Effect of horizontally shifting the center of magnetic assembly and that of keeper on the retentive force of cup-yoke type of dental magnetic attachments

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Introduction

It is well known that the retentive force of a dental magnetic attachment decreases with the presence of a vertical gap between mating surfaces, which may occur due to an air gap erroneously introduced in the magnetic assembly during the fixation of a magnetic attachment into a removable denture. Three-dimensional finite element method of analysis and some rough measurement show that even in the absence of a gap between the mating surfaces, any horizontal displacement between magnetic assembly and keeper reduces the retentive force. However, there is no available research showing the detailed relationship between the horizontal displacement and retentive force.

There are two types of magnetic attachments based on magnetic circuits: open and closed. The open magnetic circuit type has a retentive force that is directly proportional to the area of mating faces in contact; however, this type of attachment is hardly used. In contrast, the closed magnetic circuit type does not have a retentive force that is directly proportional to the contact surface area. This is due to the influence of the magnetic flux direction on the retentive force; this type of attachment is mainly used in Japanese products. A proper understanding of the retentive force change mechanism is useful for developing a magnetic attachment and the appropriate technique for clinical purposes.

Objective

In this study, the retentive forces of the cup-yoke type dental magnetic attachments were measured in different horizontal positions and the relationship between the retentive force and horizontal displacement was investigated. The mechanism leading to the changes in retentive force was studied by means of modeling.

Materials and Methods

Three dental magnetic attachments were used: GIGAUSS D600 (GC), HYPER SLIM 3513 (Morita), and an experimental set (hereafter named Hollow Keeper) based on HYPER SLIM magnetic assembly paired with a keeper (ϕ 3.5 mm) made of SUS XM27 bearing a ϕ 1.4 mm hole at the center. A measuring device that matched standard ISO 13017:2012/Amd.1:2015 was connected to a digital force gauge (ZPS, Imada). Retentive forces were measured at a crosshead speed of 5 mm/min according to standard ISO 13017 in various positions wherein the center of magnetic assembly and that of the keeper was shifted horizontally (horizontal displacement) at intervals of 100 μm.

Simplified models of the magnetic attachments were used to assist in the calculation of the contact surface area between the cup-yoke and keeper, and between the disk-yoke and keeper in different positions. The values derived from mathematical projections and calculated by formulas were compared with the actual ones.

Results

Dependencies of the retentive force on the horizontal displacement are shown in Fig. 1 for GIGAUSS, HYPER SLIM and Hollow Keeper. Retentive forces for all the three types decreases with horizontal displacement increasing. The dependency was not linear and had several inflection points. The curve lines for retentive force versus displacement for GIGAUSS and HYPER SLIM were smooth in contrast to that for Hollow Keeper, which had a single odd point slightly above 2 mm of displacement.
1. **Internal structure of dental magnetic attachments**

In order to analyze the retentive force of a magnetic attachment, it is important to know the internal structure of magnetic attachments beyond the information disclosed by the manufacturer. A simple model of magnetic attachments for illustration of internal structure is shown in Fig. 2, where \( r \) is the radius of the magnetic assembly and keeper, \( a \) is the radius of the disk-yoke, \( b \) is the thickness of the shield ring (spacer), \( c = a + b \) is the radius of the disk-yoke and thickness of shield ring, and \( d \) is the radius of the hole in the Hollow Keeper.

The magnetic attractive force \((F)\) is calculated using equation (1), where \( C \) is constant, \( B \) is the magnetic flux density, and \( S \) is the surface area. Increasing of magnetic flux density enhances the magnetic attractive force compared to a similar increasing of surface area of a mating surface.

\[
F = C \times B^2 \times S
\]  

(1)
Since in all the sets in this study a strong neodymium magnet was used, the magnetic flux density at a cup-yoke surface was predominantly saturated. Therefore, the cup-yoke surface area was the main determinant of the retentive force. Magnetic flux begins on the cup-yoke and returns to the magnet through the disk-yoke. To optimize this effect, the design should be geared toward a cup-yoke, which has a surface area \( S_{\text{cup}} \) equal or slightly smaller than that of a disk-yoke \( S_{\text{disk}} \).

\[
S_{\text{cup}} = \pi r^2 - \pi (a + b)^2 \\
S_{\text{disk}} = \pi a^2 \\
S_{\text{cup}} \leq S_{\text{disk}}
\]  

As an illustration GIGAUSS has an outer diameter of 3.6 mm (radius is 1.8 mm), and a shield ring generally has thickness of 0.15~0.2 mm. Thus \( r = 1.8 \) and \( b = 0.2 \). After substitution of the above mentioned values into equations (2-4), one obtains \( a \geq 1.169 \). The thickness of the cup-yoke \( \{r - (a + b)\} \) is 0.4 mm whereas the radius of the disk-yoke is 1.2 mm. In a similar way for HYPER SLIM \( (r = 1.75 \) and \( b = 0.2 \) one can calculate thickness of the cup-yoke as 0.4 mm and the radius of the disk-yoke as 1.15 mm.

2. Calculation of the contact surface area between magnetic assembly and keeper

Since the retentive force of magnetic attachment was dependent on the contact surface area between magnetic assembly and keeper, we calculated the contact surface area for the horizontal displacement. Horizontal displacement (\( t \)) represents the distance between the center of magnetic assembly and that of the keeper which was calculated using equation (5). The central angle \( \theta \) is formed by the intersection of two lines drawn from the center of either keeper or magnetic assembly to each of the two points where the outer margins of keeper and assembly intersects (Fig. 3a).

\[
t = 2r \cos \frac{\theta}{2}
\]  

\( t \) and \( t' \) in Fig 3a correspond to the shifting movement of the assembly or keeper. Wherein \( \theta: \pi \to 0 \) in equation (5). \( t: 0 \to 2r \). With overlap decreasing the assembly and keeper eventually become completely separated (Fig. 3a).

Two additional equations were derived in order to calculate the contact surface area of the magnetic assembly in contact with the keeper at the different positions based on displacement. Equation (6) is based on Fig. 3b, which illustrates a certain position when the area \( S_{rr} \) is formed by two components with same radii \( r \) at distance \( t \).

\[
S_{rr} = \pi r^2 \left( \frac{\theta}{2\pi} \times 2 - r \cdot r \sin \theta \right) = r^2 (\theta - \sin \theta)
\]  

Equation (7) is based on Fig. 3c, which illustrates a certain position in which the area \( S_{cr} \) was formed by two circles with different radii \( (r, c) \) at distance \( t \).

\[
S_{cr} = c^2 \theta' + r^2 \theta'' - t \times c \sin \theta' = c^2 \cos^{-1} \left( \frac{t^2 + c^2 - r^2}{2tc} \right) + r^2 \cos^{-1} \left( \frac{t^2 - c^2 + r^2}{2tr} \right) - \sqrt{4t^2c^2 - (t^2 + c^2 - r^2)^2} \frac{1}{2}
\]  

\[
\cos \theta' = \frac{t^2 + c^2 - r^2}{2tc}, \quad \cos \theta'' = \frac{t^2 - c^2 + r^2}{2tr}, \quad \sin^2 \theta' + \cos^2 \theta' = 1
\]
3. Contact surface area of GIGAUS and HYPER SLIM

Since the retentive force of the cup-yoke type magnetic attachment was relatively dependent on the area of the cup-yoke surface in contact with the keeper, the area values were calculated at three different positions of horizontal displacement as a result of shifting either the keeper or assembly laterally. Fig. 4 shows equations for calculating surface area $S_{\text{cup}1-3}$ corresponding to the horizontal displacement $t_{1-3}$.

\begin{align*}
S_{\text{cup}1} & = r^2(\theta - \sin \theta) - \pi c^2 \\
S_{\text{cup}2} & = r^2(\theta - \sin \theta) - \left\{ c^2 \cos^{-1}\left(\frac{t^2+e^2-r^2}{2tc}\right) + r^2 \cos^{-1}\left(\frac{t^2-e^2+r^2}{2tr}\right) - \frac{\sqrt{4t^2e^2-(t^2+e^2-r^2)^2}}{2} \right\} \\
S_{\text{cup}3} & = r^2(\theta - \sin \theta)
\end{align*}

(8)

(9)

(10)

Fig. 4 Schematic diagram for calculation of surface area of cup-yoke in contact with a keeper in different positions.

Values obtained from equations (2-4) for both GIGAUS and HYPER SLIM were substituted into equations (8-10), and the obtained values were plotted on graph as shown in Fig. 5a or 5b (blue lines).

Although the general view of the cup-yoke curve lines is similar to the retentive force in Fig. 1 for values less than 2 mm, the difference becomes apparent for values above 2 mm. This is due to the fact that the surface area between the cup-yoke and keeper is not the only factor that determines the functionality of a closed magnetic circuit. Therefore, dependence of the area of the disk-yoke surface in contact with keeper ($S_{\text{disk}1-3}$) on the horizontal displacement ($t_{1-3}$) was determined as shown in Fig. 6, using the following equations.

\begin{align*}
S_{\text{disk}1} & = \pi a^2 \\
S_{\text{disk}2} & = a^2 \cos^{-1}\left(\frac{t^2+a^2-r^2}{2ta}\right) + r^2 \cos^{-1}\left(\frac{t^2-a^2+r^2}{2tr}\right) - \frac{\sqrt{4t^2a^2-(t^2+a^2-r^2)^2}}{2} \\
S_{\text{disk}3} & = 0
\end{align*}

(11)

(12)

(13)

Fig. 5 Contact surface areas against the horizontal displacement

a: GIGAUS

b: HYPER SLIM
The values obtained for GIGAUS and HYPER SLIM were substituted into equations (11-13), and obtained values were plotted on a graph as shown in Fig. 5a or 5b (magenta lines).

![Fig. 6 Schematic diagram for calculation of surface areas of disk-yoke in contact with the keeper](image)

Magnetic flux from the cup-yoke passes through the keeper and returns to the magnet via the disk-yoke in a closed magnetic circuit system. When the surface area of the contact between the disk-yoke and keeper is smaller than that between the cup-yoke and keeper, the magnetic flux that exits through the cup-yoke does not return back as in case of closed magnetic circuit. In such case, the total retentive force is a function of both closed and open magnetic circuit systems. Therefore, the effect of surface area on retentive force should be considered in a relative manner. The resulting surface area related to the total magnetic circuit effect is designated as “$S_{mag}$”. When the contact surface area between the cup-yoke and keeper is smaller than that between the disk-yoke and keeper, the magnetic circuit works absolutely as a closed magnetic circuit, and $S_{mag}$ is equal to $S_{cup}$. The value of retentive force in closed magnetic circuit is 2-4 times larger than that of an open magnetic circuit. The influence area has on the retentive force in an open magnetic circuit is 1/3 of a closed magnetic circuit. Therefore $S_{mag}$ can be calculated using equation (14) in a case when the surface area of contact between the cup-yoke and keeper is larger than that between the disk-yoke and keeper.

$$S_{mag} = S_{disk} + \frac{S_{cup} - S_{disk}}{3}$$

The resulting surface area ($S_{mag}$) values were converted to retentive force. The conversion was based on $S_{mag}$ value and actual measured force value at position where $t = 0$ (without displacement), then plotted as shown in Fig. 7. The similarity in shape of curves indicates that the retentive force is dependent on the contact surface area, which influences the magnetic circuit. In general, the actual measured values were higher than those that were generated using values from projected calculations. This is probably due to leakage of magnetic flux from the cup-yoke through a portion of an area that does not contact with the keeper.

![Fig. 7 Retentive force against the horizontal displacement](image)
4. Contact surface area of Hollow Keeper

Calculation of contact surface area in case of Hollow Keeper is quite complex as illustrated in Fig. 8. The contact surface areas between magnetic assembly and keeper in various positions of displacement were plotted on a graph and shown in Fig. 9a (blue and magenta) lines for the cup-yoke and disk-yoke, respectively. It is remarkable that at certain positions of displacement, the contact surface area of either the cup-yoke or disk-yoke against keeper increased with an increase in the horizontal gap. Similarly, when the area of disk-yoke in contact with the keeper was smaller than that of the cup-yoke in contact with the keeper, the resulting surface area related to the total magnetic circuit ($S_{mag}$) was calculated using equation (14). Both derived and actual values were represented on a graph and shown in Fig. 9b. In general, the shapes of the two curves are similar. There was a peculiar point ($≈ 2.1$ mm) on the graph when the curve based on derived values showed a spike. This represents the position where the retentive force based on $S_{mag}$ increased with an increase in the horizontal gap. The spike corresponded with a previously noted odd point on the curve of actual measured values. This proves that retentive force is influenced by the resulting contact surface area related to a magnetic circuit. The reason for the large difference between actual and calculated values at $t = 1.0~2.0$ probably lies in the effect of the magnetic flux leakage through the hole that exists in the keeper in that displacement range.

Fig. 8 Schematic diagram for calculation of surface areas of cup-yoke in contact with the hollow keeper

Fig. 9 Contact surface areas and retentive force of Hollow Keeper against horizontal displacement

Conclusion

Retentive forces of all the sets decreases with increasing of horizontal displacement. The decrease was not uniform and the curve had several inflection points. Analysis carried out by means of mathematical formulas demonstrates that retentive force is influenced by the contact surface area between the cup-yoke and keeper, as well as disk-yoke and keeper, which determine the resulting contact surface area in the magnetic circuit.

References